Filter Design Report

Daniel Ayoub

March 12, 2015

Chapter 1

Low Pass Filter

1.1 Problem Selection

The main quantity required for determining which problem I was assigned is found using this expression

$$F_1 - F_2 + F_l + L_1 - L_2 - L_l$$

Substituting the letters from my name (D, A, L, A, Y, B, respectively), this expression yields $4 - 1 + 12 + 1 - 25 - 2 = -11$. Plugging this quantity into the expression

$$1 + |q| \bmod 9$$

yields question 3 as my assigned set of filter specifications.

1.2 Design Problem

My problem requires a low-pass filter design. The passband region should go up to 600 Hz, and the passband magnitude should not stray from 20 dB by more than ± 3 dB. The design asks that the filter should attenuate frequencies of greater than 9 kHz by more than 15 dB, so any frequency greater than 9 kHz should have a magnitude $\leq$ -15 dB. For the purposes of the filter design, it's useful to convert these ordinary frequency values to angular frequency values.

$$600 \, \text{Hz} \approx 3770 \, \text{rad} \, \text{s}^{-1}$$
$$9 \, \text{kHz} \approx 56.55 \, \text{krad} \, \text{s}^{-1}$$

1.3 Circuit Selection

1.3.1 First Order LPF

The first circuit examined was the following first order low-pass filter circuit.
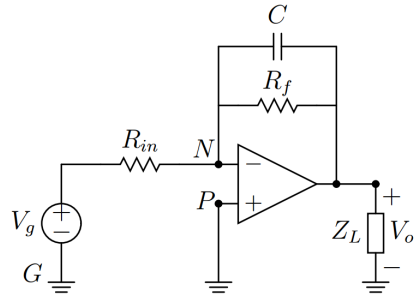
The transfer function of this circuit is given by

$$H(s) = \frac{K\omega_o}{s + \omega_o} \qquad (1.1)$$

where

$$K = -\frac{R_f}{R_{in}}$$

and

$$\omega_o = \frac{1}{R_f C}$$

Figure 1.1: First Order Low Pass Filter

ω_o is the half-power frequency, and the frequency where the magnitude will begin sloping down at -20dB/dec. Because the design specifications allow for the variation of the magnitude by $\pm$3dB, we can set this frequency equal to $3770\,\mathrm{rad\,s^{-1}}$, the end of our pass band. Furthermore, the pass band specification requires a magnitude of 20dB, so we set $K = 10$ to satisfy this property. Now, the magnitude must slope down fast enough for the circuit to successfully attenuate frequencies over 9kHz to less than -15dB.
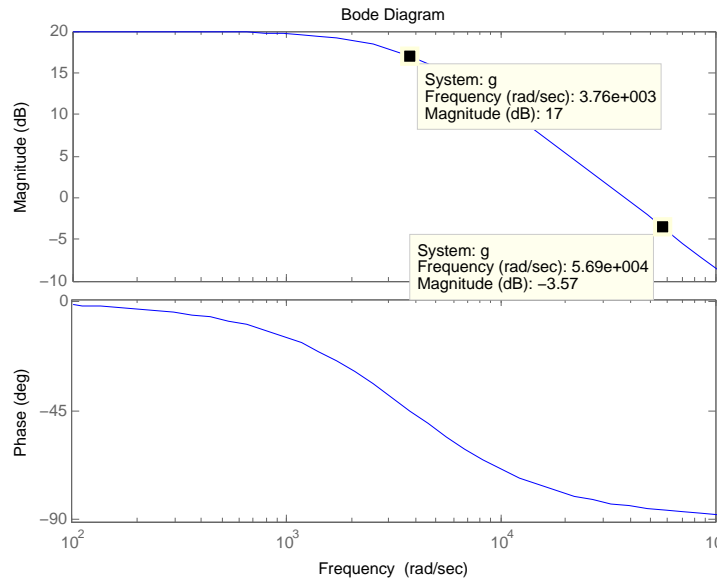

Figure 1.2: First Order Bode Plot

This magnitude response function satisfies the $\pm$3dB limit for passband frequencies, but does not decay quickly enough for frequencies above 9kHz

2

$(56.55\,\mathrm{krad\,s^{-1}})$ to have magnitude below -15dB. Furthermore, although we superficially set the gain K= 10, a closer look at the transfer function parameters shows that K is a **negative** ratio of two **resistances**, which can never be non-negative. A higher order circuit is necessary.

1.3.2 Second Order LPF

A direct second order op-amp filter would yield a negative gain, so cascading two of the first order filters tried in the last example may be feasible. This will result in a filter with the following transfer function:

$$H(s) = \left(\frac{K_1\omega_{o1}}{s+\omega_{o1}}\right)\left(\frac{K_2\omega_{o2}}{s+\omega_{o2}}\right) = \frac{K_1K_2}{\left(1+\frac{s}{\omega_{o1}}\right)\left(1+\frac{s}{\omega_{o2}}\right)} \tag{1.2}$$

where

$$K_1 = -\frac{R_{f1}}{R_{i1}}, K_2 = -\frac{R_{f2}}{R_{i2}}, \omega_{o1} = \frac{1}{R_{f1}C_1}, \omega_{o2} = \frac{1}{R_{f2}C_2}$$

The two corner frequencies allows the asymptotic bode plot to have a region with slope $-20\,\mathrm{dB\,d^{-1}}$ and a region with slope $-40\,\mathrm{dB\,d^{-1}}$. With three degrees of freedom (the two corner frequencies and the gain), this circuit is certainly able to satisfy the given specifications.

1.4 Passband Gain and Corner Frequency Determination

In choosing the corner frequencies, we have the opportunity to define a range directly after the pass band with $-20\,\mathrm{dB\,d^{-1}}$, and then increase this asymptotic slope to -40 at some higher frequency to satisfy the specifications. To try and determine the corner frequencies we need, we can first set them equal to eachother and solve an equation for an asymptote that passes through our needed frequencies. This asymptote will have slope -40 (both corner frequencies are equal so the -20 slope compounds immediately), pass through 20 dB at $f = 600\,\mathrm{Hz}$, and through -15dB at $f = 9\,\mathrm{kHz}$. The equation of this line is

$$20 = -15 - 40\log\frac{f}{9000}$$

Solving this equation for f yields the following process:

$$35 = -40\log\frac{f}{9000} \Rightarrow \frac{f}{9000} = 10^{-\frac{7}{8}} \Rightarrow f_c \approx 1.200\,\mathrm{kHz} \quad or \quad 7.540\,\mathrm{krad\,s^{-1}} \tag{1.3}$$

Now, we must write the magnitude of our original transfer function as a function of f.

$$M(f) = \frac{K_1K_2}{\sqrt{\left(1+\left(\frac{f}{f_{o1}}\right)^2\right)\left(1+\left(\frac{f}{f_{o2}}\right)^2\right)}}$$

The wanted passband gain is 20 ±3dB, so we set $K_1 K_2 = 10$ to satisfy this. Plugging in the low and high frequency and our duplicate corner frequencies and converting to dB gives us

$$M(600) = \frac{10}{\left(1 + \frac{600}{1200}^2\right)} = 8 \Rightarrow 20 \log 8 = 18.06 \, \text{dB}$$

$$M(9000) = \frac{10}{\left(1 + \frac{9000}{1200}^2\right)} = .1747 \Rightarrow 20 \log .1747 = -15.156 \, \text{dB}$$

These values are within the design constraints, but just barely. The magnitude of the high end frequency is just below the constraint of -15dB, so if we experiment around the values of these center frequencies we can achieve better results.

Trial	f_{o1}(kHz)	f_{o2}(kHz)	M_{dB} at 600Hz	M_{dB} at 9kHz	Comments
1	1.2	1.2	18.06	-15.16	Bad at low end
2	1.0	1.4	17.93	-15.40	Okay at both ends
3	1.0	1.3	17.83	-16.03	Good at both ends

Thus, values of $f_{o1} = 1 \, \text{kHz}$ and $f_{o2} = 1.3 \, \text{kHz}$ can be used.

1.5   Component Value Selection

To choose resistor values for the circuit, we examine the necassry passband gain and what it is equal to in terms of circuit components:

$$K = \frac{R_{f1}}{R_{i1}} * \frac{R_{f2}}{R_{i2}} \tag{1.4}$$

If we set the R_i values equal to $1 \, \text{k}\Omega$, we can set the R_f values to the available $1.8 \, \text{k}\Omega$ and $5.6 \, \text{k}\Omega$ and achieve a gain $K = 10.08$

Try 1: Using $\omega_{o1} = 2000\pi$ and $R_{f1} = 1.8 \, \text{k}\Omega$, we get $C_1 \approx 8.842 \times 10^{-8} \, \text{F}$. If we use the standard capacitor value of $0.082 \, \mu\text{F}$ we get $\omega_{o1} \approx 6.613 \, \text{krad s}^{-1} \approx 1.052 \, \text{kHz}$. Using $\omega_{o2} = 2600\pi$ and $R_{f2} = 5.6 \, \text{k}\Omega$, we get $C_2 \approx 2.186 \times 10^{-8} \, \text{F}$. If we use the standard capacitor value of $0.022 \, \mu\text{F}$ we get $\omega_{o2} \approx 8.117 \, \text{krad s}^{-1} \approx 1.291 \, \text{kHz}$. These component values give us the following magnitudes:

f_{o1}(kHz)	f_{o2}(kHz)	Gain	M_{dB} at 600Hz	M_{dB} at 9kHz
1.052	1.291	10.08	17.928	-15.658

Thus, the full set of parameter values are:
$R_{f1} = 1.8 \, \text{k}\Omega$, $R_{i1} = 1 \, \text{k}\Omega$ and $C_1 = 0.082 \, \mu\text{F}$
$R_{f2} = 5.6 \, \text{k}\Omega$, $R_{i2} = 1 \, \text{k}\Omega$ and $C_2 = 0.022 \, \mu\text{F}$
$K = 10.08$, $f_{o1} = 1.052 \, \text{kHz}$,   and $f_{o2} = 1.291 \, \text{kHz}$

1.6 Transfer Function

The final transfer function can be determined by plugging the parameters found in section 1.5 into eq. (1.2):

$$H(s) = \frac{10.08}{\left(1 + \frac{s}{6613}\right)\left(1 + \frac{s}{8117}\right)} \tag{1.5}$$
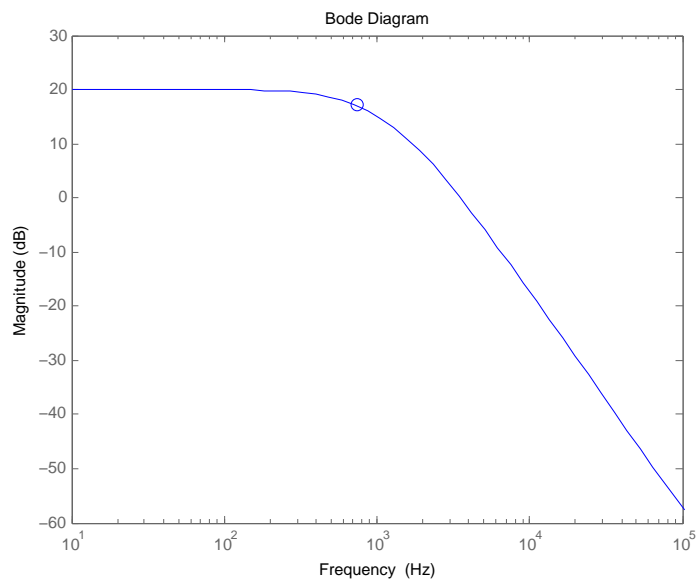
1.7 Bode Diagram
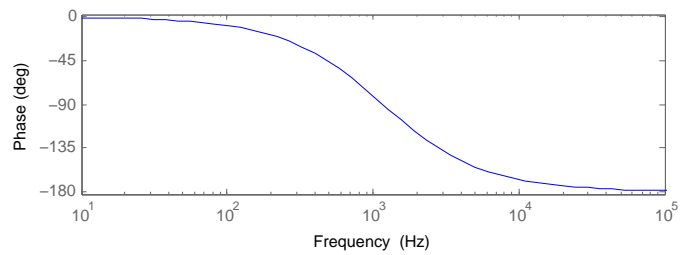

Figure 1.3: Magnitude Response Characteristic

Figure 1.4: Phase Response Characteristic

The plotted magnitude and phase response functions show that the designed filter satisfies the required specifications. The plot also shows the half power frequency $f_c \approx 742.8\,\text{Hz}$, which is circled on figure 1.3.

1.8   Sensitivity Plot

We can do a rudimentary sensitivity analysis by arbitrarily altering the values of varios circuit components to partially simulate the value tolerances of real-world circuit components. In this basic study, we will alter R_{i1} and C_2 in the circuit by 5% of their nominal value. Their new values will be

$$R_{i1} = 950\,\Omega, \quad and \quad C_2 = 0.0209\,\mu\text{F}$$

This change yields this altered gain and these corner frequencies:

$$K = 10.61, \quad \omega_{c2} = 8544.1\,\text{rad s}^{-1}, \quad f_{c2} = 1.360\,\text{kHz}$$
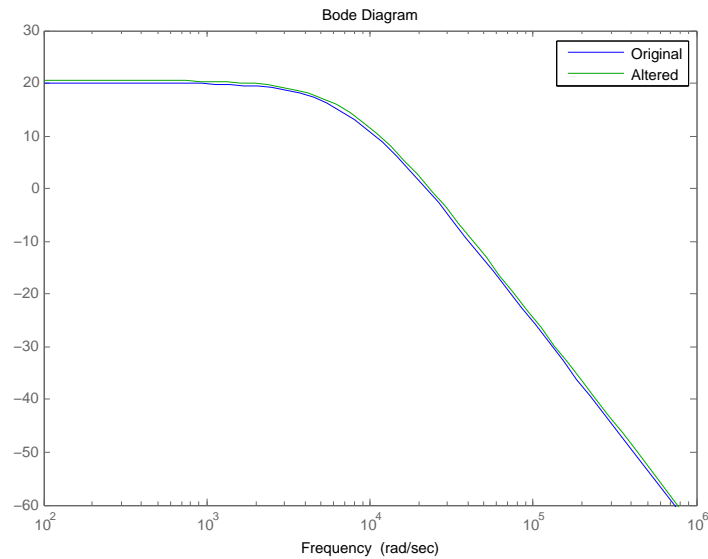
And the magnitude response graphed in figure 1.5.

Figure 1.5: Magnitude Response Characteristic

Chapter 2

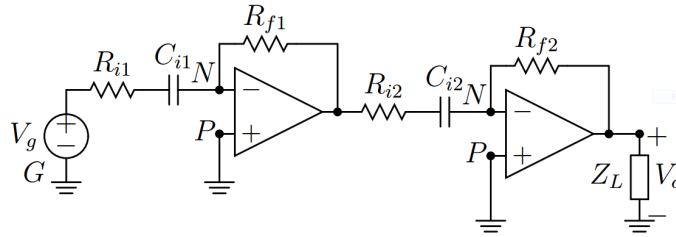# High Pass Filter

2.1 Transfer Function Determination

Figure 2.1: Second Order High Pass Filter

In section 1.7, we determined the half power frequency of our low-pass filter to be

$$f_c = 742.8\,\text{Hz} \quad \omega_c = 4.667\,\text{krad s}^{-1}$$

Now, we may use this frequency and our original transfer function to derive a high pass filter from our original circuit. We perform the following substitution:

$$s \Rightarrow \frac{\omega_c^2}{s}$$

$$H(s) = \frac{K}{\left(1 + \frac{s}{\omega_{o1}}\right)\left(1 + \frac{s}{\omega_{o2}}\right)} \Rightarrow H(s) = \frac{Ks^2}{\left(s + \frac{\omega_c^2}{\omega_{o1}}\right)\left(s + \frac{\omega_c^2}{\omega_{o2}}\right)} \tag{2.1}$$

Where $K = 10.08$, $\omega_{o1} = 6.613\,\text{krad s}^{-1}$, and $\omega_{o2} = 8.117\,\text{krad s}^{-1}$. This transfer function can be fulfilled by a second order high pass filter, shown in the figure 2.1.
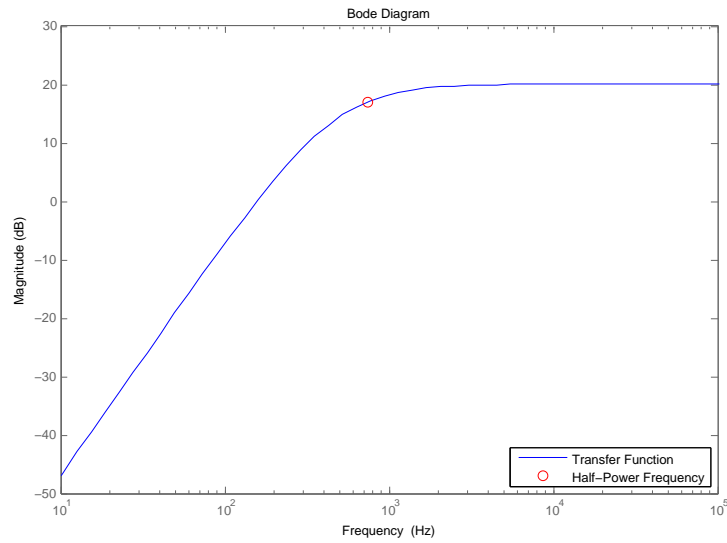
2.2   Transfer Function Plot

Figure 2.2: Magnitude Response Characteristic of High Pass Filter

This transfer function can be plotted using MATLAB, similarly to the low-pass filter function. Plotting a point at the original cutoff frequency at 17dB reveals that the original cutoff frequency is identical to the new cutoff frequency.

Chapter 3

Scaled Low Pass Filter

3.1 Cutoff Frequency Scaling

The design specifications require a 3dB frequency 10 times that of the prior 3dB frequency. The new cutoff frequency should be

$$f_c = 7.428\,\text{kHz} \quad \omega_c = 46.67\,\text{krad s}^{-1}$$

Now, we use the following scaling equations:

$$C' = \frac{C}{K_m K_f}, \quad R' = K_m R$$

Where K_m and K_f are the impedance and frequency scaling constants, respectively. $K_f = 10$, because we require a new cutoff frequency 10 times higher than our pre-scaled circuit. Our original capacitance values aren't too small, so we have room to decrease them further if necessary. Because of this, we can set $K_m = 1$, to keep the resistances constant and decrease the capacitances by a factor of 10 in total. This gives us new component values:

$R_{f1} = 1.8\,\text{k}\Omega, \quad R_{i1} = 1\,\text{k}\Omega \quad \text{and} \quad C_1 = 8.2\,\text{nF}$
$R_{f2} = 5.6\,\text{k}\Omega, \quad R_{i2} = 1\,\text{k}\Omega \quad \text{and} \quad C_2 = 2.2\,\text{nF}$
$K = 10.08, \qquad f_{c1} = 10.783\,\text{kHz}, \quad \text{and} \quad f_{c2} = 12.918\,\text{kHz}$

3.2 Frequency Response

Plugging these values into our low pass filter transfer function and computing the bode plot using MATLAB gives us the magnitude response plot in figure 3.1. The circled frequency is the new $f_c \approx 7.530\,\text{kHz}$. This is approximately 10 times the cutoff frequency of the original low-pass filter.
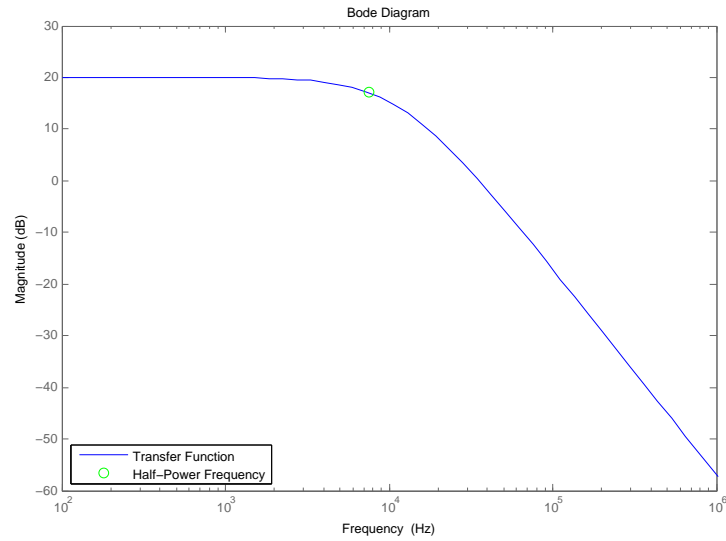

Figure 3.1: Magnitude Response Characteristic of Scaled Filter

Conclusion

This extensive filter design project solidified my theoretical understanding of filters. I achieved results that were within the design constaints after extensively manipulating transfer functions and translating theoretical equations into real-life resistance and capacitance values. The low-pass filter schematic contained in this report is a concise and feasible solution to the specified design problem. Working on this project helped to familiarize myself with various aspects of filter design, including the several iterative processes that allowed me to feel how altering certain component values would affect the filter's frequency response characteristic. Furthermore, the sensitivity analysis, although rudimentary, showed that real-life errors in component values play an important role in comprehensive feasibility studies of filter designs. Finally, the two filter transforms performed at the end of this report showed two powerful methods of manipulating a transfer function and extracting component values to derive two entirely different filter circuits and frequency response characteristics. This filter design project offered great first experience into the realm of real-world Electrical Engineering.

Appendix A

Matlab Code

A.1 Component Guess-And-Check Calculator

```
fc1 = 1052;
fc2 = 1291;
low = 600;
high = 9000;
f = @(x) 10./sqrt((1+(x./fc1).^2).*(1+(x./fc2).^2));
g = @(y) 20*LOG10(f(y));
g(low)
g(high)
```

A.2 Low Pass Filter Calculations

```
K = 10.08;
w1 = 6613;
w2 = 8117;
wo = sqrt(w1*w2);
beta = w1+w2;
transfer = tf(K*wo^2,[1 beta wo^2]);
bodemag(transfer)
[magLow, phaseLow] = bode(transfer, 600*2*pi);
[magHigh, phaseHigh] = bode(transfer, 9000*2*pi);
magLowdB = 20*log10(magLow)
magHighdB = 20*log10(magHigh)
wc = bandwidth(transfer)
fc = wc / (2*pi);
plot(fc, 20*log10(10.08)-3, 'bo');
```

A.3 High Pass Filter Calculations

```
K = 10.08;
w1 = 6613;
w2 = 8117;
wc = 4662;
fc = wc / (2*pi)
wo = sqrt(wc^4/(w1*w2));
beta = wc^2/w1 + wc^2/w2;
transfer = tf([K 0 0],[1 beta wo^2]);
bodemag(transfer, 'b')
wc2 = bandwidth(transfer)
hold on
plot(fc, 20*log10(10.08)-3, 'ro');
legend('Transfer Function', 'Half-Power Frequency', 'Location', 'se');
```

A.4 Scaled Filter Calculations

```
K = 10.08;
w1 = 81168.8311;
w2 = 67750.677;
wo = sqrt((w1*w2));
beta = w1+w2;
transfer = tf(K*wo^2,[1 beta wo^2]);
bodemag(transfer)
wc = bandwidth(transfer)
fc = wc / (2*pi)
hold on
plot(fc, 20*log10(10.08)-3, 'go');
legend('Transfer Function', 'Half-Power Frequency', 'Location', 'sw');
```